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SELECTED METHODS OF DETERMINING PROBABILISTIC POWER FLOW

Summary: Taking into consideration the variability of the future conditions determining the branches load, it might be interesting to use probabilistic power flow for solving power flow. To determine the probabilistic power flow analytical methods (including approximation) or simulation methods may be applied. The choice of calculation method is the result of a compromise between the quality of the results obtained and the required time of calculations. In this paper, we have characterized cumulates method and point estimation method (these are analytical methods). We have cited Polish research initializing these issues (convolution method and the method of voltage orthogonal components). Existing possibilities of probabilistic power flow implementation issues are discussed in terms of available software.

Keywords: probabilistic power flow, transmission grid, development planning

WYBRANE METODY WYZNACZANIA PROBABILISTYCZNEGO ROZPŁYWU MOCY

Streszczenie. Ze względu na zmienność przyszłych warunków wyznaczania obciążeń gałęziowych ciekawym rozwiązaniem jest wykorzystanie probabilistycznego rozptywu mocy. Do wyznaczenia probabilistycznego rozptywu mocy mogą być zastosowane metody analityczne (w tym aproksymacyjne) lub symulacyjne. Wybór metody obliczeniowej jest wynikiem kompromisu pomiędzy jakością uzyskanych wyników oraz niezbędnym czasem obliczeń. Wśród metod analitycznych w artykule scharakteryzowano metodę kumulant oraz estymacji punktowej, wskazując przy tym polskie prace inicjujące opisywaną problematykę (metodę splotu funkcji oraz metodę składowych prostokątnych napięć węzłowych). W zakresie dostępnego oprogramowania wskazano istniejące możliwości implementacji zagadnienia probabilistycznego rozptywu mocy.

Słowa kluczowe: probabilistyczny rozptyw mocy sieć przesyłowa, planowanie rozwoju

1. INTRODUCTION

Probabilistic methods and models in power engineering have been researched in Poland for many decades [1,3,4,5,9,10,13,15]. The scope of research in most cases has not included the problem of probabilistic power flow (PRM). Some exceptions may be found in [3,15], where analytic methods of calculating PRM have been proposed and in [1], where method based on simulation has been described. Probabilistic methods are often used abroad for planning development of transmission grid; however, the extent to which such methods are used is highly diverse [6,11]. Among external reasons for their use we may point out uncertainty and indeterminacy of the data on future development conditions, including those caused by evolution of electrical energy market, ownership changes in power engineering sector, expansion of renewable energy sources (incorporating distributed generations), changes in structure of power consumption related to economic growth (in the broadest sense).

Methods of determining PRM may be classified as simulation, analytical and approximation ones. Analytical and approximation methods are often combined into one class defined as analytical methods [6,8]. Probabilistic power flow is defined as the flow, where input data and calculation results assume the form of probability distribution (as parameters characteristic of such functions). Input data is analogous to data present in deterministic power flow: the quantities are power demand, availability and power generation determined for nodal points. Moreover, topology of grid circuit is also treated as input data. Calculation results are obtained as probability distributions of node voltages and branch flows. Technical parameters of the grid circuit (admittances, susceptancies etc.) may be handled as variables given in the form of probability distributions. However, usually in e.g. development calculations they are assumed to be constant and time-invariant. This is a simplifying assumption. Structure (topology) of grid circuit, ascertained as availability of its different components, is a random variable with two-state distribution: a given element may be either on or off. Failure rate of a given element (corrected as per planned outages) is a characteristic parameter of this distribution.

2. SELECTED ANALYTICAL METHODS

In general, publication [3] is indicated as the one initializing PRM issue. In this work function convolutions were used to determine probability distributions of power flows in lines. This approach required some simplifying assumptions, adopting the linear dependence of active power flows in lines on active power demand in nodal points and

independence of random changes of power demand in different grid nodes. The discussed work used dc model of the network. Next step in the development of analytical methods was a proposal of constructing non-linear probabilistic node equations for ordinary moments and, alternately, for central moments [15]. The essence of the method lay in using probabilistic flow equations taking into account orthogonal components of node voltages; the only simplification assumed was that these components may be described by Gaussian distribution. Number of probabilistic equations indispensable to solving this problem depends (in this case) on the square of number of analysed grid nodal points; for instance, for models of national power grid used nowadays (c. 4400 nodal points) this number is close to 38 million.

Another group of methods employed for determining PRM uses a cumulant concept. Cumulants are determined for different types of probability distributions of input and output data. These methods also used diverse procedures for reconstructing probability distributions. In accordance with proposition propounded in [12,19], additional random variable z is created; it is a linear combination of n independent random variables X_i (see (1)). Next, function $\Phi_z(s)$ is constructed. This function generates ordinary moments in accordance with (2); another function generates cumulants in accordance with (3). It may be observed that function generating cumulants is constructed by taking a logarithm of function generating ordinary moments. In order to determine moment or cumulant of k^{th} order, it is necessary to calculate derivative of A^{th} order of the function generating moments or cumulants with respect to s , respectively, and then to substitute $s = 0$. Cumulant of k^{th} order A^* may then be expressed as (4).

$$z = a_1 x_1 + a_2 x_2 + \dots + a_n x_n \quad (1)$$

$$\Phi_z(s) = E(e^{jsz}) = E(e^{jsa_1 x_1}) E(e^{jsa_2 x_2}) \dots E(e^{jsa_n x_n}) \quad (2)$$

$$\Psi_z(s) - \ln \Phi_z(s) = \Psi_{x_1}(a_1 s) + \Psi_{x_2}(a_2 s) + \dots + \Psi_{x_n}(a_n s) \quad (3)$$

$$\Lambda_{z,k} = \frac{d^k \Psi_z(0)}{ds^k} = a_1^k \Psi_{x_1}^{(k)}(0) + a_2^k \Psi_{x_2}^{(k)}(0) + \dots + a_n^k \Psi_{x_n}^{(k)}(0) \quad (4)$$

The subsequent stage of this method is introduction of cumulants into flow problem. The unknown cumulants of resultant random variables are determined on the basis of cumulants of input variables. To this end, Hessian of Lagrange function is used; it defines coefficients of linear transformation of the cumulants in accordance with dependencies (5) and (6). In other words, the unbalance vector (of the error) e.g. in Newton method of determining the power flow is replaced with vector of cumulants of active and reactive

$$\text{power demands at network nodes} - H^{-1} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{bmatrix} \quad (5)$$

$$\Lambda_{z_i,k} = a_{i,1}^k \Lambda_{x_1,k} + a_{i,2}^k \Lambda_{x_2,k} + \cdots + a_{i,n}^k \Lambda_{x_n,k} \quad (6)$$

The last element of cumulant method is reconstruction of probability distribution of resultant random variable on the basis of calculated cumulants. When cumulant order is increased, then accuracy of distribution reconstruction is higher. In order to reproduce probability distributions, Gram-Chalier or Edgeworth series are used. Theorem stating that any probability distribution may be expressed with a series in accordance with (7) is then used. This series is constructed out of c_j coefficients corresponding to moments (Gram-Chalier) or cumulants (Edgeworth) of Chebyshev-Hermite polynomial $He_j(x)$ and characteristic function of Gaussian distribution $\varphi(x)$. Chebyshev-Hermite polynomial factor of k^{th} order is generated using general relationships given in (8). Therefore, any arbitrary probability distribution of resultant variable may be described by (9).

$$f(x) = \sum_{j=0}^{\infty} c_j He_j(x) \varphi(x) \quad (7)$$

$$He_k(x) = (-1)^k \frac{d^k \alpha(x)}{d\alpha^k} \quad \text{oraz} \quad \alpha(x) = e^{-(1/2)x^2} \quad (8)$$

$$f(x) = \left[1 + \frac{\Lambda_{x,3}}{3!} He_3(x) + \frac{\Lambda_{x,4}}{4!} He_4(x) + \frac{\Lambda_{x,5}}{5!} He_5(x) + \frac{(\Lambda_{x,6} + 10\Lambda_{x,3}^2)}{6!} He_6(x) + \cdots \right] \varphi(x) \quad (9)$$

The basic flaw of the method described above lies in not accounting for the impact of availability of network infrastructure (i.e. outages) on the obtained results. Solution of this problem is presented in [7], where compensation method is proposed for simulating line disconnections. After calculating power flow, line disconnection is simulated by fictitious injections of power in the nodes at both sides of disconnected line; then branch flows are corrected in accordance with linear dependence of power flow on fictitious node powers. The list of switched-off elements is in this case predefined and it may contain both single and multiple outages. The end result in a given simulation is obtained using general relationships for calculation of conditional probability. It must be remembered that discussed emergency states do not fill entire space of possible system states; therefore, probabilities of different states must be corrected in such a way that their total should be equal to one.

The basic assumption of cumulant method is the independence of random variables which are the input data for calculating power flow. However, in reality we usually

encounter correlated variables, for instance quantity of power generated by the wind farm depends on wind velocity, which is influenced by locality, but it may affect several or more network nodes. That is why method of orthogonalization of input data has been proposed in [17]; orthogonalization here means that these data are written down as sums of independent partial random variables. Identical partial random variables may occur at different network nodes. With respect to power generated by wind farms this boils down to simulating (sampling) wind velocity in a given region first; on this basis quantity of node power generation is determined. The same author indicates the need for corrections in cumulant method (actually this means correcting the resultant probability distributions), when identification of given number of active limitations in power system is considered (these lead to results highly deviating from average values). This relates to specified and critical states of power system. They are not very probable, but their impact on end results is significant.

Other methods for PRM determination are classed as point estimate methods. Point estimate method was proposed by Emilio Rosenblueth in 1975. In successive years it was developed by different authors in the range of functionalities characterised in Table 1 [6,8,14]. Point estimate scheme focuses on statistical information provided by several first central moments of input data, determined for so-called concentration points (see Table 1 - K is number of concentration points, while n is number of independent input variables). Each concentration point is defined by the pair: locality ξ and weight factor w .

Table 1

Characteristic of selected point estimate methods [8]

Author of the method	Number of simulations	Effectiveness of application in large systems	Possibility of accounting for	
			Correlated variables	Asymmetric variables
Rosenblueth	2 ⁿ	Very low	yes	yes
Li	n^3	low	yes	yes
Harr	2 ⁿ	high	yes	no
Hong	Kn ;	high	no	yes

Let $y = f(x_1, x_2, \dots, x_n)$ be resultant variable determined by any function of n independent random variables. In Hong method and for scheme $(K-1)n+1$, variable y is evaluated $(K-1)n$ times; it is presumed that different variables assume average values and one of them assumes value p_{xkl} determined for concentration point; this is expressed by relationships (10) and (11). Moreover, value of this function is determined for the case when all input data assume average values. In view of this, when $K=3$, number of equations to be solved is equal to $2n+1$, and locality of third concentration point is the same as that

of average value, i.e. $\xi_{x_k,3} = 0$.

$$y = f(\mu_{x_1}, \mu_{x_2}, \dots, p_{x_{1,l}}, \dots, \mu_{x_n}) \quad (10)$$

$$p_{x_{k,l}} = \mu_{x_k} + \xi_{x_{k,l}} \sigma_{x_k} \quad (11)$$

Location of concentration points and corresponding weights are determined by solving non-linear set of equations described with (12). Sum of weight coefficients for all variables is equal to $1/n$, while sum of the products of weight coefficients and location of concentration points to the j^{th} power results in central moment of j^{th} order λ_{j,x_k} of variable x_k . Solution of equation (12) in the form of relationships determining locations of concentration points and weight coefficients for scheme $2n+1$ is expressed with formulas (13) and (14). It must be noted that locations of concentrations points and weight coefficients depend on central moments of 3rd and 4th order, i.e. on skewness and kurtosis of probability distribution of given random variable (input variable). In case of methods proposed by other authors listed in Table 1 this dependency does not occur.

$$\begin{cases} \sum_{l=1}^K w_{x_{k,l}} = \frac{1}{n} \\ \sum_{l=1}^K w_{x_{k,l}} (\xi_{x_{k,l}})^j = \lambda_{j,x_k}, \quad j = 1, \dots, 2K-1 \end{cases} \quad (12)$$

$$\begin{cases} \xi_{x_{k,l}} = \frac{\lambda_{3,x_k}}{2} + \sqrt{\lambda_{4,x_k} - \frac{3}{4} \lambda_{3,x_k}^2} \\ \xi_{x_{k,2}} = \frac{\lambda_{3,x_k}}{2} - \sqrt{\lambda_{4,x_k} - \frac{3}{4} \lambda_{3,x_k}^2} \\ \xi_{x_{k,3}} = 0 \end{cases} \quad (13)$$

$$w_{x_{k,1}} = \frac{1}{\xi_{x_{k,1}}^2 - \xi_{x_{k,1}} \xi_{x_{k,2}}}; \quad w_{x_{k,2}} = \frac{1}{\xi_{x_{k,2}}^2 - \xi_{x_{k,1}} \xi_{x_{k,2}}}; \quad w_{x_{k,3}} = \frac{1}{n} - \frac{1}{\lambda_{4,x_k} - \lambda_{3,x_k}^2} \quad (14)$$

After establishing all concentration points and their weight for input data and after calculating the system function for a number of times (depending on a given scheme), it is possible to determine average value of resultant variable and standard deviation as well, in accordance with relationships (15) to (17). Reproduction of the distribution may be done using Gram-Charlier or Edgeworth series (in the same way as in case of cumulant method).

$$\mu_y = e(y) \cong \sum_{k=1}^n \sum_{l=1}^n w x_{k,l} f(\mu_{x_1}, \mu_{x_2}, \dots, p_{x_{k,l}}, \dots, \mu_{x_n}) \quad (15)$$

$$E(y^2) \cong \sum_{k=1}^n \sum_{l=1}^n w x_{k,l} [f(\mu_{x_1}, \mu_{x_2}, \dots, p_{x_{k,l}}, \dots, \mu_{x_n})]^2 \quad (16)$$

$$\sigma_y = \sqrt{E(y^2) - \mu_y^2} \quad (17)$$

Limitations in the use of point estimation method are related to want of possibility of taking into account the probability density distributions of variables which cannot be described by average values and standard deviations (Bernoulli distribution). This concerns mostly variables related to availability of network infrastructure or availability of power units. In case of dependent variables this method does not yield correct results either.

3. SIMULATION METHODS

Simulation methods for calculating PRM are based on execution of a specific number of deterministic power flows; data for the flow comes from simulation of input data in accordance with their probability distributions. To determine input data values for n^{th} simulation, different simulation methods may be applied such as (among others) Monte Carlo method, layer methods, adaptation methods etc. Number of indispensable simulations depends on adopted simulation method; for Monte Carlo method it may vary from several hundreds to several tens of thousands depending on the size of analysed system. Other methods e.g. layer method make it possible to decrease required number of simulations.

In case of large network systems, after conducting PRM calculations we may expect results in the form of Gaussian distributions. In accordance with Lyapunov Central Limit Theorem (CLT) [5] the probability distribution of sum of independent random variables when number of components is high and regardless of what probability distribution these variables are subjected to, converges into Gaussian distribution with probability density expressed by formula (18). None of the random variables may be, however, dominant. Such dominant character of random variable may occur in case of power unit prevailing in a given region or in case of major customer of electrical energy, or in case of outages of network infrastructure critical elements.

$$p(x) = \frac{1}{\sqrt{2\pi\sigma_x}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} \quad (18)$$

The problem of pseudorandom number generator is neglected in publications in case of calculations for network systems containing several, several tens or several thousand network nodes. This issue might be significant in particular when large number of simulations is required. Among pseudorandom number generators we may distinguish congruential generators, chaotic oscillators and those based on shift registers [16]. From the viewpoint of PRM result accuracy, the quality of representing probability distribution of random variable is significant. On account of the fact, that development calculations are conducted on multi-annual basis (which is specifically burdened with some indeterminacy), it may be assumed that impact of generally applied pseudorandom number generators on the quality of obtained results is negligible. However, this issue should be investigated further.

4. COMPUTATIONAL TOOLS

The overview of available programs [1,2,11,18,20], conducted from the perspective of PRM feasibility, may be concluded by categorizing these programs into three general classes:

- a) software dedicated to probabilistic power flow,
- b) software containing some probabilistic elements or making allowance for them due to in-built programming languages,
- c) software where it is possible to execute probabilistic flow only if the autonomous power flow program is carried out repeatedly.

As far as software dedicated to probabilistic power flow is contemplated, four programs have been identified: UC-OPF-GRS, PLF, PRA and ASSESS. All these programs make it possible to determine probabilistic power flow by simulation method. UC-OPF-GRS program is a principal (computationally) component of the withdrawn PRiMSP software platform which has been elaborated for PSE S.A. in the beginning of 21st century. Its name is derived from main functions of the program: selection of power units (Unit Commitment – UC), optimal power flow (OPF) and generator of random states (GRS). This program was written as a series of MATLAB-based functions. Random state generator used Monte Carlo method as well as Latin Hypercube Sampling. PLF and PRA programs were worked out by EPRI also around 2000. PLF program made it possible to carry out simulations with Monte Carlo method; changes in power demand, power generation and availability of network infrastructure were taken into consideration in this program. PRA program made it possible to calculate specific probabilistic indicators related to line or transformer overload, voltage, voltage stability and power supply restrictions set on the customers. The last program (ASSESS) is used nowadays; it has been elaborated by RTE and National Grid Transco.

This software makes it possible to model different states of power engineering system in a random or systematised way. Randomness may be related to any variable occurring in the planning process such as: power generation by a wind turbine, power demand level, operational mode of element (emergency and operational shutdowns of devices). During typical analysis from 1000 to 30000 random states are generated by Monte Carlo method. Each of these states may be subjected to full-range analysis available on this platform.

The most abundant category of computational tools is the one containing programs with specific probabilistic elements or containing programming language which make it possible to include probabilistic elements. We may list the following programs among those best known: PowerFactory, MatPower, Neplan, PS SE, PowerWord, Plexos, SimPow, Scope, Plans. These programs are usually supplied with in-built high-level programming languages such as Python or DSL; it is usually possible to create macro commands to control all or selected functions of the program; they may also be written using MATLAB functions. It must be emphasized that as far as we know, PowerFactory is the only program, which uses both simulation method (Monte Carlo) and analytical method (two-point estimation method) for calculating PRM. Some of the mentioned programs are also characterized by the built-in ability of dividing computational problem among computers accessible via LAN-network; this is significant from PRM viewpoint, since it substantially decreases computational time.

5. CONCLUSION

Choice of simulation or analytical of approximation method for calculating PRM is in reality dictated by compromise between quality of obtained results and required computational time. Publications on the subject indicate that simulation methods are most accurate; they however require longer computational times than other methods. Computational time is directly influenced by: number of analysed development options (i.e. reinforcements of network system), model size (number of nodes, branches, generators), flow model (dc or ac), range of single flow problem (selection of generating units, optimal power flow) and number of simulations. In view of present technological progress, where computers with computing powers exceeding 1,2PFLOPS (i.e. more than 1.2×10^{15} floating-point operations per second) are available in the country, total computational time is no longer a leading criterion for choosing calculation method. However, companies responsible for planning the development of the network do not employ such computers and, besides, power flow software is not adapted to utilise computational capacity of supercomputers.

Accuracy of analytical or approximation methods is another issue (compared to simulation methods, which are generally considered to be most accurate). Comparisons presented in publications usually relate mostly to test systems (models) containing several tens or at most slightly more than one hundred nodes. The obtained results should therefore be verified for real network systems containing from several to even several tens of thousands of network nodes. In case of most popular analytical methods characterized in current paper, the maximum error of determining average value (for voltages, branch flows) did not exceed several per cent, while for variance it did not exceed 10%. In case of point estimation method such accuracy was ensured by described scheme $2n+1$, other methods listed in Table 1 do not guarantee such computational accuracy.

It must be emphasised that no instance of impact of PRM determination method selection on possible investment decision in planning process has been identified in known publications. On the one hand, the problem may be particularly important in case of limited financial means. On the other hand, development calculations are usually conducted for several or more than ten so-called scenarios of development conditions. The probability of their occurrence is judged by the experts, i.e. a rough estimate only is provided. Moreover, the uncertainty of future input data, e.g. power demand information together with potential probability distributions may end in getting results with accuracy comparable to that obtained by analytical and approximation methods, which is quite satisfactory.

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